Summary

Dynamics of thin functionally graded cylindrical shells - tolerance modelling

The subject-matter of this doctoral thesis is the analytical modelling and analysis of dynamic problems for thin linearly elastic Kirchhoff-Love-type open circular cylindrical shells having ***a functionally graded macrostructure*** and ***a tolerance-periodic microstructure*** in circumferential direction. It means that ***on the microscopic level***, the shells consist of a very large number of separated, small elements regularly distributed along circumferential direction and perfectly bonded to each other (or to the homogeneous matrix). These elements, called ***cells***, are treated as thin shells. It is assumed that ***the adjacent cells are nearly identical*** (*i.e. they have nearly the same geometrical, elastic and inertial properties*), ***but the distant elements can be very different.*** At the same time, the shells have constant structure in axial direction. ***On the microscopic level***, the geometrical, elastic and inertial properties of these shells are determined by highly oscillating, non-continuous and *tolerance-periodic functions* in circumferential direction. On the other hand, ***on the macroscopic level***, the averaged properties of the shells are described by functions being ***continuous and slowly varying*** along the direction of tolerant periodicity. It means that the tolerance-periodic shells under consideration can be treated as made of ***functionally graded materials*** (FGM) and called ***functionally graded shells.*** Moreover, since macroscopic (i.e. averaged) properties of the shells are graded in direction normal to interfaces between constituents, this gradation is referred to as ***the transversal gradation.***

Dynamic behaviour of such shells are described by the known governing equations of Kirchhoff-Love theory for thin elastic shells. For tolerance-periodic shells, coefficients of these equations are highly oscillating, non-continuous and tolerance-periodic functions. That is why the direct application of these equations to investigations of specific dynamic problems is non-effective even using computational methods.

The first aim of the doctoral thesis has been to formulate and

discuss three <u>new</u> mathematical averaged models with continuously slowly-varying coefficients constituting a proper tool for the analysis of selected dynamic problems in the thin cylindrical shells with a tolerance-periodic microstructure and transversally graded macrostructure in the circumferential direction. Moreover, two from these models take into account the effect of a microstructure size on the dynamic shell behaviour. In order to formulate these models, the tolerance, consistent asymptotic and combined asymptotic-tolerance modelling procedures, cf. [Woźniak Cz. *et al.* (eds.): *Mathematical modelling and analysis in continuum mechanics of microstructured media.* Silesian University of Technology Press, Gliwice 2010], ***have been applied to the starting Euler-Lagrange equations, which explicit form coincides with the governing equations of Kirchhoff-Love theory for thin elastic shells.***

The ***tolerance approach*** is based on the notion of ***tolerance relations*** between points and real numbers related to the accuracy of the performed measurements and calculations. Tolerance relations are determined by ***tolerance parameters.*** Other fundamental concepts of this modelling technique are those of ***slowly-varying functions, tolerance-periodic functions, fluctuation shape functions and averaging operation.*** The tolerance modelling is based on two assumptions. The first of them is called ***the tolerance averaging approximation*** and makes it possible to neglect terms of an order of tolerance parameters. The second one is termed ***the micro-macro decomposition.*** It states that the displacement fields occurring in the starting lagrangian have to be *the tolerance-periodic functions* in the direction of tolerant periodicity. Hence, they can be decomposed into ***unknown averaged displacements (macrodisplacements) being slowly-varying functions and fluctuations represented by finite series of products of the known highly oscillating continuous tolerance-periodic fluctuation shape functions and unknown slowly-varying fluctuation amplitudes.***

The tolerance modelling technique applied to the starting Euler-Lagrange equations has been realized in two steps. The first step has been based on ***the tolerance averaging of the starting lagrangian*** by applying the ***micro-macro decomposition***, the ***averaging operation*** as well as ***the tolerance averaging approximation.*** In the second step, using *the principle of stationary action* to ***the averaged action functional*** defined by means of the ***tolerance-averaged lagrangian***, we have arrived at the averaged Euler-Lagrange equations and then at their explicit form given by the constitutive relations and the dynamic balance equations. Summarizing, ***the tolerance model for the analysis of dynamic problems for thin linearly elastic Kirchhoff-Love-type cylindrical shells having a transversally graded macrostructure and***

a tolerance-periodic microstructure in circumferential direction is represented by the constitutive relations and the dynamic balance equations together with the micro-macro decomposition and the physical reliability conditions. The basic unknowns are ***macrodisplacements*** and ***fluctuation amplitudes*** which must be slowly-varying along x-coordinate parametrizing the shell midsurface in the circumferential direction. ***The resulting tolerance model equations have coefficients which are continuous and slowly-varying in the direction of tolerant periodicity. Moreover, some of these coefficients depend on microstructure size. The length-scale effect can be analysed not only in dynamic but also in stationary problems.***

On passing from tolerance averaging to the consistent asymptotic averaging, the concept of highly oscillating fluctuation shape functions is retained only. The notions of tolerance-periodic functions and slowly-varying functions are not introduced. The fundamental assumption imposed on the starting lagrangian in the framework of this approach is called ***the consistent asymptotic decomposition.*** It states that the displacement fields occurring in the lagrangian have to be replaced by families of fields depending on parameter $\varepsilon \in (0,1]$ and defined in an arbitrary cell. These families of displacements are decomposed into averaged part described by unknown functions (macrodisplacements) being continuously bounded in the tolerant periodicity direction and highly-oscillating part depending on ε. This highly-oscillating part is represented by the known highly-oscillating fluctuation shape functions multiplied by unknown functions (fluctuation amplitudes) being continuously bounded in the direction of tolerant periodicity.

Asymptotic modelling procedure applied to the starting Euler-Lagrange equations has been realized in two steps. The first step has been ***the consistent asymptotic averaging of the starting lagrangian under the consistent asymptotic decomposition.*** In the second step, applying *the principle of stationary action* to *the consistent asymptotic action functional* defined by means of the asymptotically averaged lagrangian, we have arrived at the asymptotically averaged Euler-Lagrange equations and then at their explicit form. Finally, after eliminating unknown fluctuation amplitudes, we have obtained the ***asymptotic model equations*** expressed only in ***macrodisplacements. Coefficients in the asymptotic equations are continuously slowly variable in x, but they are independent of the microstructure cell size.*** Thus, ***contrary to the tolerance model, the consistent asymptotic one is not able to describe the length-scale effect on the overall shell dynamics.***

The combined asymptotic-tolerance model for the analysis of selected dynamic problems in the functionally graded shells under consideration ***has***

been formulated by applying the combined modelling procedure given in [Woźniak Cz. *et al.* (eds.): *Mathematical modelling and analysis in continuum mechanics of microstructured media.* Silesian University of Technology Press, Gliwice 2010]. This combined modelling includes the consistent asymptotic and the tolerance non-asymptotic modelling techniques which are combined together into a new procedure. ***The equations of combined model proposed here consist of the asymptotic (macroscopic) model equations formulated by means of the consistent asymptotic procedure and having continuous and slowly changing coefficients independent of a microstructure length and of the superimposed tolerance (microscopic) model equations derived by applying the tolerance modelling technique and having continuous and slowly-varying coefficients depending also on a cell size. Both the models are combined together under assumption that in the framework of the asymptotic model the solutions to the problem under consideration are known.*** It has been shown *that under special condition imposed on the fluctuation shape functions,* ***the combined model makes it possible to separate the macroscopic description of some special problems from their microscopic description.*** Thus, ***an important advantage of this model is that it allows us to study micro-dynamics of the shells under consideration independently of their macro-dynamics.***

Solutions to selected initial/boundary value problems formulated in the framework of the tolerance model and the microscopic part of combined model have a physical sense only if unknowns of the aforementioned models are slowly-varying functions in the direction of tolerant periodicity. Moreover, these conditions can be also used for the *a posteriori* evaluation of tolerance parameters and hence, for the verification of the physical reliability of the obtained solutions.

The second aim of this doctoral thesis has been to apply the tolerance and asymptotic models derived here to evaluation of the length-scale effect in some special problems dealing with free vibrations of the tolerance-periodic shells under consideration. In order to find analytical solutions to the governing equations of these models (equations with continuous and slowly-varying coefficients), the known Ritz variational method has been applied. It has been shown that ***in the framework of the tolerance model, not only the fundamental cell-independent lower, but also the new additional higher-order cell-dependent free vibration frequencies can be derived and analysed. The higher free vibration frequencies cannot be determined applying asymptotic models commonly used for investigations of dynamics of the periodic or tolerance-periodic shells.*** It has been shown that the differences between the fundamental lower free vibration frequencies derived from the tolerance model and free vibration frequencies obtained from the asymptotic one are ***negligibly small.*** Thus, *the effect of the microstructure size on the fundamental lower free vibration frequencies of the shells under consideration can be neglected.* Hence, ***the asymptotic model being more simple than the tolerance non-asymptotic one is sufficient from the point of view of calculations made for the vibration problems under consideration.***

The third aim of the dissertation has been to apply the microscopic equations derived in the second step of the combined asymptotic-tolerance modelling to the analysis of length-scale effect in some special problems dealing with the shell micro-vibrations and with the long wave propagation related to micro-fluctuations of the shell displacements. These equations are independent of solutions obtained in the framework of the consistent asymptotic model (i.e. model derived in the first step of combined modelling) ***and make it possible to analyse selected problems of the shell micro-dynamics independently of the shell macro-dynamics. This is the greatest advantage of the proposed combined model.*** Moreover, these micro-dynamic equations involve terms with time and spatial derivatives of unknown micro-fluctuation amplitudes. Hence, they describe certain time-boundary layer and space-boundary layer phenomena strictly related to the specific form of initial and boundary conditions imposed on unknown fluctuation amplitudes.

It has been evidenced that using these micro-dynamic equations, ***the new cell-dependent higher free vibration frequencies can be determined and analysed <u>independently</u> of the fundamental, classical cell-independent lower free vibration frequencies.*** Since the micro-dynamic equations contain continuously slowly-varying coefficients, the known Galerkin method was used to obtain approximate formulas of free micro-vibration frequencies.

Some new important results have been obtained analysing the harmonic micro-vibrations with vibration frequency $\breve{\omega}$. It has been shown that the form of these micro-vibrations depends on relations between values of vibration frequency $\breve{\omega}$ and a certain new additional higher-order free vibration frequency $\breve{\omega}_*$ depending on the cell size. ***The micro-vibrations can decay exponentially. They can decay linearly. For certain interrelations between $\breve{\omega}$ and $\breve{\omega}_*$ we deal with a non-decayed form of micro-vibrations (micro-vibrations oscillate) or with resonance micro-vibrations. Moreover, it has been shown that the micro-dynamic equations of the combined model describe the space-boundary layer phenomena.***

Some new important results have been obtained analysing the long wave propagation problem related to micro-fluctuations in axial direction. It was shown that *the tolerance-periodic micro-heterogeneity of the* *shells leads to exponential waves and to dispersion effects.* Moreover, *the new wave propagation speed depending on the microstructure size has been obtained.*

All the above length-scale problems studied within the micro-dynamic equations of the combined model cannot be analysed in the framework of the asymptotic models commonly used for investigations of dynamic behaviour of the cylindrical shells with a functionally graded macrostructure and a tolerance-periodic microstructure.

The functionally graded shells being objects of considerations in this doctoral thesis are widely applied in civil engineering, most often as roof girders and bridge girders.

The results obtained in the dissertation generate new directions of further investigations. The anticipated directions of investigations can be related to: the modelling of stationary and dynamic stability problems in the framework of linear Kirchhoff-Love second-order theory, the non-linear shell dynamics and stability, the modelling of dynamic thermoelasticity problems and others.